Exam. Code : 211001
Subject Code : 5475

M.Sc. (Mathematics) I^{st} Semester
MECHANICS—I
Paper—MATH-554

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Attempt **TWO** questions from each unit. Each question carries equal marks.

UNIT—I

I. Find the velocity and acceleration of a particle moving along a curve. In the usual notations, show that $\frac{d\vec{t}}{ds} = \kappa \vec{n}$.

II. Determine the components of acceleration of a particle moving along the curve $r = ae^{b\theta}$ such that the radius vector moves with constant angular velocity ω.

III. Define vector angular velocity. In the usual notations show that $\vec{\omega} = \frac{1}{2} \operatorname{curl} \vec{V}$.

IV. If $\frac{d}{dt}$ and $\frac{\partial}{\partial t}$ denote the rate of change relative to fixed frame and moving frame with angular velocity ω respectively, then for any vector $\vec{F}$ show that

$$\frac{d\vec{F}}{dt} = \frac{\partial \vec{F}}{\partial t} + \vec{w} \times \vec{F}$$

and hence find the velocity and acceleration.

UNIT—II

V. A body of mass M, travelling in a straight line, is supplied with power P and is subjected to a resistance Mkv^2, where v is the speed and k is a constant. Prove that the speed of the body cannot exceed a certain value and that, if it starts from rest, it acquires half the maximum speed after travelling a distance $\frac{1}{3k}\log\frac{8}{7}$.

VI. What do you mean by conservative force ? Give example. Show that for a single particle moving in a conservative field of force, the sum of kinetic and potential energy is constant.

VII. A particle of mass m is constrained to execute a rectilinear SHM under a force towards O of magnitude $m\omega^2 x$, x being the particle's displacement from O. When passing through O its velocity is V and when its velocity has become half of V in the same direction, an impulse I is applied to the particle in the direction of its motion. Assuming the same law of force, find the time and total distance travelled from O to the first position of instantaneous rest.

VIII. A particle is projected upward with a velocity V in a medium whose resistance varies as the square of the velocity. Discuss the motion.

UNIT—III

IX. A fixed wire is in the shape of a cardioid $r = a(1 + \cos\theta)$, the initial line being the downward vertical. A small ring of mass m can slide on the wire and is attached to the point $r = 0$ of the cardioids by an elastic string of natural length 'a' and modulus 4 mg. If the particle is released from rest when the string is horizontal, show that

$$a\dot{\theta}^2 (1 + \cos\theta) - g\cos\theta(1 - \cos\theta) = 0.$$

X. A particle is projected with velocity 'u' in a direction inclined at an angle α to the horizontal. Determine the horizontal and vertical displacement after time t on the assumption that gravity is the only force acting. Show that path of trajectory is a parabola.

XI. Discuss the motion of a particle of mass m, moving on the smooth inner surface of the paraboloid of revolution : $x^2 + y^2 = 4az$, whose axis is vertical and vertex downward.

XII. What is a cycloid ? Show that its equation is $s = 4a \sin\psi$ in usual notations. A particle slides down a smooth cycloid whose axis is vertical and vertex downward. Find the velocity of the particle and reaction on it at any point of the cycloid.

UNIT—IV

XIII. Derive the equation of motion of the orbit of a particle moving under central force in terms of reciprocal polar coordinates.

XIV. Show that the inverse square law of force directed towards a fixed point always produces a conic type orbit.

XV. Discuss the motion of a particle moving in an elliptic orbit under the inverse square law of attraction and subjected to a small blow in the tangential direction.

XVI. State Kepler's laws of planetary motion. Two gravitating particles A and B of mass 'm' and 'M' respectively, move under the force of their mutual attraction. If the orbit of A relative to B is a circle of radius 'a' described with velocity v, show that $v = \sqrt{\gamma(M+m)/a}$.

UNIT—V

XVII. Define principal axes a product of inertia. Show that the products of inertia with respect to principal axes are zero.

XVIII. What do you mean by equimomental systems ? State and prove necessary and sufficient conditions for the two systems to be in equimomental.

XIX. Find the moment of inertia of a rigid body about a line having direction cosines $< \lambda, \mu\ \nu >$. Let the rigid body is rotating about this line with angular velocity ω, then find the expression of kinetic energy of the body in terms of its moment of inertia.

XX. State perpendicular axis theorem. Use it to find the moment of inertia of an elliptic disc about a line perpendicular to the plane of the disc.